13th Edition

CALCULUS

for Business, Economics, Life Sciences, and Social Sciences

Barnett   Ziegler   Byleen

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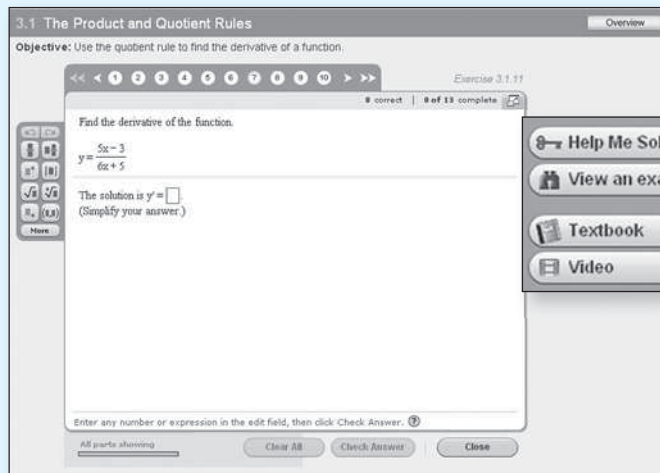
Breaks the problem into manageable steps. Students enter answers along the way.

Reviews a problem like the one assigned.

Links to the appropriate section in the textbook.

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CALCULUS

FOR BUSINESS, ECONOMICS, LIFE SCIENCES, AND SOCIAL SCIENCES

Thirteenth Edition

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CONTENTS

Available separately: Calculus Topics to Accompany Calculus, 13e,
and College Mathematics, 13e

PREFACE

The thirteenth edition of *Calculus for Business, Economics, Life Sciences, and Social Sciences* is designed for a one-term course in Calculus for students who have had one to two years of high school algebra or the equivalent. The book's overall approach, refined by the authors' experience with large sections of college freshmen, addresses the challenges of teaching and learning when prerequisite knowledge varies greatly from student to student.

The authors had three main goals when writing this text:

▶ To write a text that students can easily comprehend

▶ To make connections between what students are learning and how they may apply that knowledge

▶ To give flexibility to instructors to tailor a course to the needs of their students.

Many elements play a role in determining a book's effectiveness for students. Not only is it critical that the text be accurate and readable, but also, in order for a book to be effective, aspects such as the page design, the interactive nature of the presentation, and the ability to support and challenge all students have an incredible impact on how easily students comprehend the material. Here are some of the ways this text addresses the needs of students at all levels:

▶ Page layout is clean and free of potentially distracting elements.

▶ *Matched Problems* that accompany each of the completely worked examples help students gain solid knowledge of the basic topics and assess their own level of understanding before moving on.

▶ Review material (Appendix A and Chapter 1) can be used judiciously to help remedy gaps in prerequisite knowledge.

▶ A *Diagnostic Prerequisite Test* prior to Chapter 1 helps students assess their skills, while the *Basic Algebra Review* in Appendix A provides students with the content they need to remediate those skills.

▶ *Explore and Discuss* problems lead the discussion into new concepts or build upon a current topic. They help students of all levels gain better insight into the mathematical concepts through thought-provoking questions that are effective in both small and large classroom settings.

▶ Instructors are able to easily craft homework assignments that best meet the needs of their students by taking advantage of the variety of types and difficulty levels of the exercises. Exercise sets at the end of each section consist of a ***Skills Warm-up*** (four to eight problems that review prerequisite knowledge specific to that section) followed by problems divided into categories A, B, and C by level of difficulty, with level-C exercises being the most challenging.

▶ The MyMathLab course for this text is designed to help students help themselves and provide instructors with actionable information about their progress. The immediate feedback students receive when doing homework and practice in MyMathLab is invaluable, and the easily accessible e-book enhances student learning in a way that the printed page sometimes cannot.

Most important, all students get substantial experience in modeling and solving real-world problems through application examples and exercises chosen from business and economics, life sciences, and social sciences. Great care has been taken to write a book that is mathematically correct, with its emphasis on computational skills, ideas, and problem solving rather than mathematical theory.

Finally, the choice and independence of topics make the text readily adaptable to a variety of courses (see the chapter dependencies chart on page xi). This text is one of three books in the authors' college mathematics series. The others are *Finite Mathematics for Business, Economics, Life Sciences, and Social Sciences*, and *College Mathematics for Business, Economics, Life Sciences, and Social Sciences*; the latter contains selected content from the other two books. *Additional Calculus Topics*, a supplement written to accompany the Barnett/Ziegler/Byleen series, can be used in conjunction with any of these books.

New to This Edition

Fundamental to a book's effectiveness is classroom use and feedback. Now in its thirteenth edition, *Calculus for Business, Economics, Life Sciences, and Social Sciences* has had the benefit of a substantial amount of both. Improvements in this edition evolved out of the generous response from a large number of users of the last and previous editions as well as survey results from instructors, mathematics departments, course outlines, and college catalogs. In this edition,

▶ The Diagnostic Prerequisite Test has been revised to identify the specific deficiencies in prerequisite knowledge that cause students the most difficulty with calculus.

▶ Chapters 1 and 2 of the previous edition have been revised and combined to create a single introductory chapter (Chapter 1) on functions and graphs.

▶ Most exercise sets now begin with a *Skills Warm-up*—four to eight problems that review prerequisite knowledge specific to that section in a just-in-time approach. References to review material are given in the answer section of the text for the benefit of students who struggle with the warm-up problems and need a refresher.

▶ Section 6.4 has been rewritten to cover the trapezoidal rule and Simpson's rule.

▶ Examples and exercises have been given up-to-date contexts and data.

▶ Exposition has been simplified and clarified throughout the book.

▶ An Annotated Instructor's Edition is now available, providing answers to exercises directly on the page (whenever possible). *Teaching Tips* provide less-experienced instructors with insight on common student pitfalls, suggestions for how to approach a topic, or reminders of which prerequisite skills students will need. Lastly, the difficulty level of exercises is indicated only in the instructor's edition so as not to discourage students from attempting the most challenging "C" level exercises.

▶ *MyMathLab* for this text has been enhanced greatly in this revision. Most notably, a "Getting Ready for Chapter X" has been added to each chapter as an optional resource for instructors and students as a way to address the prerequisite skills that students need, and are often missing, for each chapter. Many more improvements have been made. See the detailed description on pages xiv and xv for more information.

Trusted Features

Emphasis and Style

As was stated earlier, this text is written for student comprehension. To that end, the focus has been on making the book both mathematically correct and accessible to students. Most derivations and proofs are omitted, except where their inclusion adds significant insight into a particular concept as the emphasis is on computational skills, ideas, and problem solving rather than mathematical theory. General concepts and results are typically presented only after particular cases have been discussed.

Design

One of the hallmark features of this text is the **clean, straightforward design** of its pages. Navigation is made simple with an obvious hierarchy of key topics and a judicious use of call-outs and pedagogical features. We made the decision to maintain a two-color design to

help students stay focused on the mathematics and applications. Whether students start in the chapter opener or in the exercise sets, they can easily reference the content, examples, and *Conceptual Insights* they need to understand the topic at hand. Finally, a functional use of color improves the clarity of many illustrations, graphs, and explanations, and guides students through critical steps (see pages 22, 75, and 306).

Examples and Matched Problems

More than 300 completely worked examples are used to introduce concepts and to demonstrate problem-solving techniques. Many examples have multiple parts, significantly increasing the total number of worked examples. The examples are annotated using blue text to the right of each step, and the problem-solving steps are clearly identified. **To give students extra help** in working through examples, dashed boxes are used to enclose steps that are usually performed mentally and rarely mentioned in other books (see Example 4 on page 9). Though some students may not need these additional steps, many will appreciate the fact that the authors do not assume too much in the way of prior knowledge.

EXAMPLE 2   Tangent Lines   Let $f(x) = (2x - 9)(x^2 + 6)$.

(A) Find the equation of the line tangent to the graph of $f(x)$ at $x = 3$.

(B) Find the value(s) of x where the tangent line is horizontal.

SOLUTION

(A) First, find $f'(x)$:

$$f'(x) = (2x - 9)(x^2 + 6)' + (x^2 + 6)(2x - 9)'$$
$$= (2x - 9)(2x) + (x^2 + 6)(2)$$

Then, find $f(3)$ and $f'(3)$:

$$f(3) = [2(3) - 9](3^2 + 6) = (-3)(15) = -45$$
$$f'(3) = [2(3) - 9]2(3) + (3^2 + 6)(2) = -18 + 30 = 12$$

Now, find the equation of the tangent line at $x = 3$:

$$y - y_1 = m(x - x_1) \quad y_1 = f(x_1) = f(3) = -45$$
$$y - (-45) = 12(x - 3) \quad m = f'(x_1) = f'(3) = 12$$
$$y = 12x - 81 \quad \text{Tangent line at } x = 3$$

(B) The tangent line is horizontal at any value of x such that $f'(x) = 0$, so

$$f'(x) = (2x - 9)2x + (x^2 + 6)2 = 0$$
$$6x^2 - 18x + 12 = 0$$
$$x^2 - 3x + 2 = 0$$
$$(x - 1)(x - 2) = 0$$
$$x = 1, 2$$

The tangent line is horizontal at $x = 1$ and at $x = 2$.

Matched Problem 2 Repeat Example 2 for $f(x) = (2x + 9)(x^2 - 12)$.

Each example is followed by a similar *Matched Problem* for the student to work while reading the material. This actively involves the student in the learning process. The answers to these matched problems are included at the end of each section for easy reference.

Explore and Discuss

Most every section contains *Explore and Discuss* problems at appropriate places to encourage students to think about a relationship or process before a result is stated or to investigate additional consequences of a development in the text. This serves to foster critical thinking and communication skills. The Explore and Discuss material can be used for in-class discussions or out-of-class group activities and is effective in both small and large class settings.

New to this edition, annotations in the instructor's edition provide tips for less-experienced instructors on how to engage students in these Explore and Discuss activities, expand on the topic, or simply guide student responses.

Explore and Discuss 1   Let $F(x) = x^2$, $S(x) = x^3$, and $f(x) = F(x)S(x) = x^5$. Which of the following is $f'(x)$?

(A) $F'(x)S'(x)$ (B) $F(x)S'(x)$

(C) $F'(x)S(x)$ (D) $F(x)S'(x) + F'(x)S(x)$

Exercise Sets

The book contains over 4,500 carefully selected and graded exercises. Many problems have multiple parts, significantly increasing the total number of exercises. Exercises are paired so that consecutive odd- and even-numbered exercises are of the same type and difficulty level. Each exercise set is designed to allow instructors to craft just the right assignment for students. Exercise sets are categorized as Skills Warm-up (review of pre-requisite knowledge), and within the Annotated Instructor's Edition only, as A (routine, easy mechanics), B (more difficult mechanics), and C (difficult mechanics and some theory) to make it easy for instructors to create assignments that are appropriate for their classes. The *writing exercises*, indicated by the icon ✎, provide students with an opportunity to express their understanding of the topic in writing. Answers to all odd-numbered problems are in the back of the book.

Applications

A major objective of this book is to give the student substantial experience in modeling and solving real-world problems. Enough applications are included to convince even the most skeptical student that mathematics is really useful (see the Index of Applications at the back of the book). Almost every exercise set contains application problems, including applications from business and economics, life sciences, and social sciences. An instructor with students from all three disciplines can let them choose applications from their own field of interest; if most students are from one of the three areas, then special emphasis can be placed there. Most of the applications are simplified versions of actual real-world problems inspired by professional journals and books. No specialized experience is required to solve any of the application problems.

Additional Pedagogical Features

The following features, while helpful to any student, are particularly helpful to students enrolled in a large classroom setting where access to the instructor is more challenging or just less frequent. These features provide much-needed guidance for students as they tackle difficult concepts.

▶ **Call-out boxes** highlight important definitions, results, and step-by-step processes (see pages 57, 63–64).

▶ **Caution statements** appear throughout the text where student errors often occur (see pages 82, 274, and 332).

> ⚠ **CAUTION** The expression $0/0$ does not represent a real number and should never be used as the value of a limit. If a limit is a $0/0$ indeterminate form, further investigation is always required to determine whether the limit exists and to find its value if it does exist. ▲

▶ **Conceptual Insights**, appearing in nearly every section, often make explicit connections to previous knowledge, but sometimes encourage students to think beyond the particular skill they are working on and attain a more enlightened view of the concepts at hand (see pages 56–57, 116, and 306).

> **CONCEPTUAL INSIGHT**
>
> Rather than list the points where a function is discontinuous, sometimes it is useful to state the intervals on which the function is continuous. Using the set operation **union,** denoted by $\cup$, we can express the set of points where the function in Example 1 is continuous as follows:
>
> $$(-\infty, -4) \cup (-4, -2) \cup (-2, 1) \cup (1, 3) \cup (3, \infty)$$

▶ The newly revised **Diagnostic Prerequisite Test**, located at the front of the book, provides students with a tool to assess their prerequisite skills prior to taking the course. The **Basic Algebra Review**, in Appendix A, provides students with seven sections of content to help them remediate in specific areas of need. Answers to the Diagnostic Prerequisite Test are at the back of the book and reference specific sections in the Basic Algebra Review or Chapter 1 for students to use for remediation.

Graphing Calculator and Spreadsheet Technology

Although access to a graphing calculator or spreadsheets is not assumed, it is likely that many students will want to make use of this technology. To assist these students, optional graphing calculator and spreadsheet activities are included in appropriate places. These include brief discussions in the text, examples or portions of examples solved on a graphing calculator or spreadsheet, and exercises for the student to solve. For example, linear and quadratic regression are introduced in Section 1.3, and regression techniques on a graphing calculator are used at appropriate points to illustrate mathematical modeling with real data. All the optional graphing calculator material is clearly identified with the icon ⊟ and can be omitted without loss of continuity, if desired. Optional spreadsheet material is identified with the icon ▦. Graphing calculator screens displayed in the text are actual output from the TI-84 Plus graphing calculator.

Chapter Reviews

Often it is during the preparation for a chapter exam that concepts gel for students, making the chapter review material particularly important. The chapter review sections in this text include a comprehensive summary of important terms, symbols, and concepts, keyed to completely worked examples, followed by a comprehensive set of Review Exercises. Answers to Review Exercises are included at the back of the book; *each answer contains a reference to the section in which that type of problem is discussed* so students can remediate any deficiencies in their skills on their own.

Chapter Dependencies

Diagnostic
Prerequisite Test

PART ONE: A LIBRARY OF ELEMENTARY FUNCTIONS*

1 Functions and Graphs

PART TWO: CALCULUS

2 Limits and
the Derivative

3 Additional
Derivative Topics

4 Graphing and
Optimization

6 Additional
Integration Topics

5 Integration

7 Multivariable
Calculus

8 Trigonometric
Functions

APPENDIXES

A Basic Algebra Review

B Special Topics
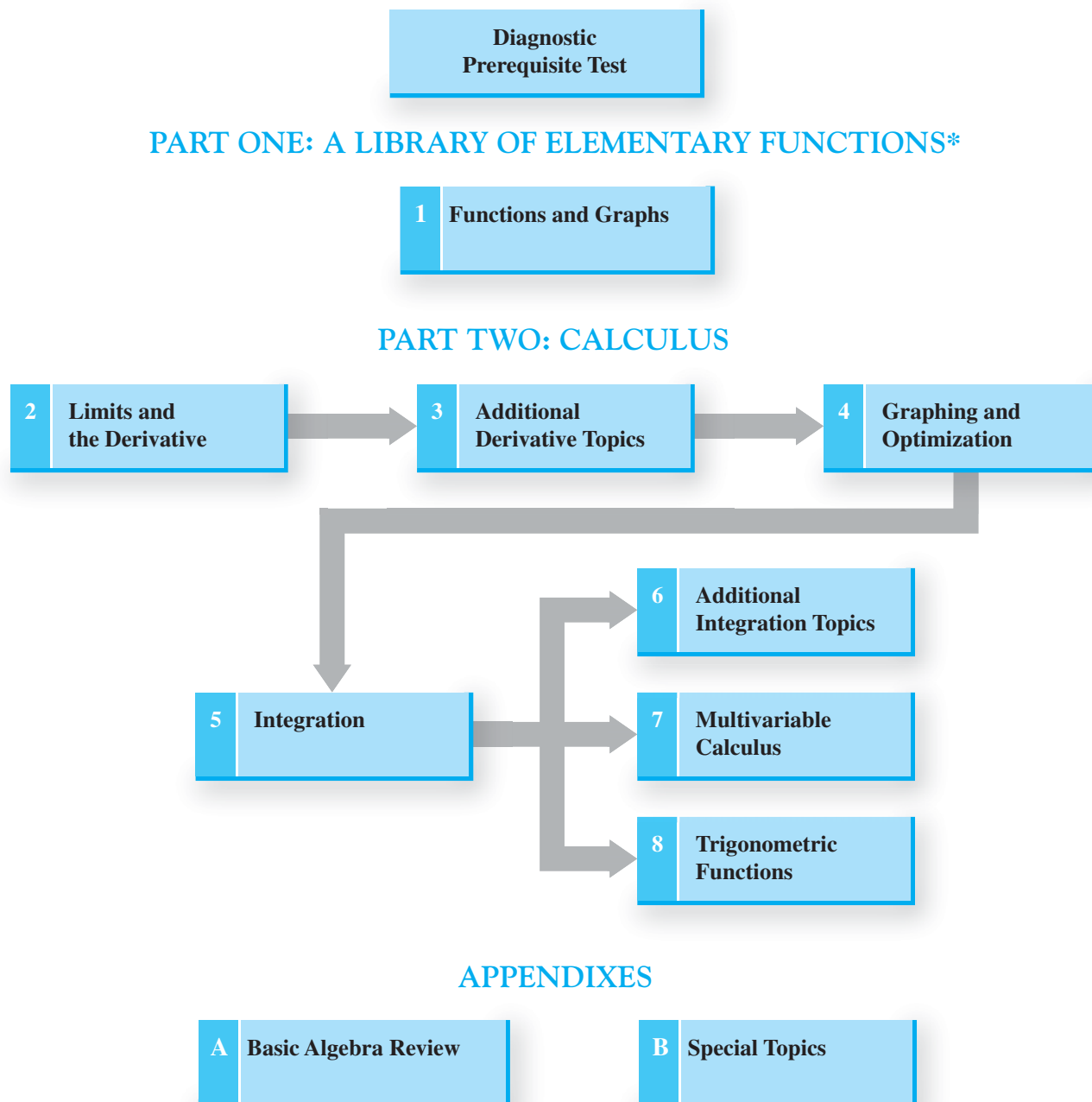
*Selected topics from Part One may be referred to as needed in
Part Two or reviewed systematically before starting Part Two.

Content

The text begins with the development of a library of elementary functions in **Chapter 1**, including their properties and applications. Many students will be familiar with most, if not all, of the material in this introductory chapter. Depending on students' preparation and the course syllabus, an instructor has several options for using the first chapter, including the following:

 (i) Skip Chapter 1 and refer to it only as necessary later in the course;

(ii) Cover Section 1.3 quickly in the first week of the course, emphasizing price–demand equations, price–supply equations, and linear regression, but skip the rest of Chapter 1;

(iii) Cover Chapter 1 systematically before moving on to other chapters.

The material in Part Two (Calculus) consists of differential calculus (Chapters 2–4), integral calculus (Chapters 5 and 6), multivariable calculus (Chapter 7), and a brief discussion of differentiation and integration of trigonometric functions (Chapter 8). In general, Chapters 2–5 must be covered in sequence; however, certain sections can be omitted or given brief treatments, as pointed out in the discussion that follows (see the Chapter Dependencies chart on page xi).

▶ **Chapter 2** introduces the derivative. The first three sections cover limits (including infinite limits and limits at infinity), continuity, and the limit properties that are essential to understanding the definition of the derivative in Section 2.4. The remaining sections of the chapter cover basic rules of differentiation, differentials, and applications of derivatives in business and economics. The interplay between graphical, numerical, and algebraic concepts is emphasized here and throughout the text.

▶ In **Chapter 3** the derivatives of exponential and logarithmic functions are obtained before the product rule, quotient rule, and chain rule are introduced. Implicit differentiation is introduced in Section 3.5 and applied to related rates problems in Section 3.6. Elasticity of demand is introduced in Section 3.7. The topics in these last three sections of Chapter 3 are not referred to elsewhere in the text and can be omitted.

▶ **Chapter 4** focuses on graphing and optimization. The first two sections cover first-derivative and second-derivative graph properties. L'Hôpital's rule is discussed in Section 4.3. A graphing strategy is presented and illustrated in Section 4.4. Optimization is covered in Sections 4.5 and 4.6, including examples and problems involving end-point solutions.

▶ **Chapter 5** introduces integration. The first two sections cover antidifferentiation techniques essential to the remainder of the text. Section 5.3 discusses some applications involving differential equations that can be omitted. The definite integral is defined in terms of Riemann sums in Section 5.4 and the fundamental theorem of calculus is discussed in Section 5.5. As before, the interplay between graphical, numerical, and algebraic properties is emphasized. These two sections are also required for the remaining chapters in the text.

▶ **Chapter 6** covers additional integration topics and is organized to provide maximum flexibility for the instructor. The first section extends the area concepts introduced in Chapter 6 to the area between two curves and related applications. Section 6.2 covers three more applications of integration, and Sections 6.3 and 6.4 deal with additional methods of integration, including integration by parts, the trapezoidal rule, and Simpson's rule. Any or all of the topics in Chapter 6 can be omitted.

▶ **Chapter 7** deals with multivariable calculus. The first five sections can be covered any time after Section 4.6 has been completed. Sections 7.6 and 7.7 require the integration concepts discussed in Chapter 5.

▶ **Chapter 8** provides brief coverage of trigonometric functions that can be incorporated into the course, if desired. Section 8.1 provides a review of basic trigonometric concepts. Section 8.2 can be covered any time after Section 4.3 has been completed. Section 8.3 requires the material in Chapter 5.

▶ **Appendix A** contains a concise review of basic algebra that may be covered as part of the course or referenced as needed. As mentioned previously, **Appendix B** contains additional topics that can be covered in conjunction with certain sections in the text, if desired.

Accuracy Check

Because of the careful checking and proofing by a number of mathematics instructors (acting independently), the authors and publisher believe this book to be substantially error free. If an error should be found, the authors would be grateful if notification were sent to Karl E. Byleen, 9322 W. Garden Court, Hales Corners, WI 53130; or by e-mail to kbyleen@wi.rr.com.

Student Supplements

Student's Solutions Manual

▶ By Garret J. Etgen, University of Houston

▶ This manual contains detailed, carefully worked-out solutions to all odd-numbered section exercises and all Chapter Review exercises. Each section begins with Things to Remember, a list of key material for review.

▶ ISBN-13: 978-0-321-93173-3

Additional Calculus Topics to Accompany Calculus, 13e, and College Mathematics, 13e

▶ This separate book contains three unique chapters: Differential Equations, Taylor Polynomials and Infinite Series, and Probability and Calculus.

▶ ISBN 13: 978-0-321-93169-6; ISBN 10: 0-321-931696

Graphing Calculator Manual for Applied Math

▶ By Victoria Baker, Nicholls State University

▶ This manual contains detailed instructions for using the TI-83/TI-83 Plus/TI-84 Plus C calculators with this textbook. Instructions are organized by mathematical topics.

▶ Available in MyMathLab.

Excel Spreadsheet Manual for Applied Math

▶ By Stela Pudar-Hozo, Indiana University–Northwest

▶ This manual includes detailed instructions for using Excel spreadsheets with this textbook. Instructions are organized by mathematical topics.

▶ Available in MyMathLab.

Guided Lecture Notes

▶ By Salvatore Sciandra,
Niagara County Community College

▶ These worksheets for students contain unique examples to enforce what is taught in the lecture and/or material covered in the text. Instructor worksheets are also available and include answers.

▶ Available in MyMathLab or through Pearson Custom Publishing.

Videos with Optional Captioning

▶ The video lectures with optional captioning for this text make it easy and convenient for students to watch videos from a computer at home or on campus. The complete set is ideal for distance learning or supplemental instruction.

▶ Every example in the text is represented by a video.

▶ Available in MyMathLab.

Instructor Supplements

New! Annotated Instructor's Edition

▶ This book contains answers to all exercises in the text on the same page as the exercises whenever possible. In addition, Teaching Tips are provided for less-experienced instructors. Exercises are coded by level of difficulty only in the AIE so students are not dissuaded from trying more challenging exercises.

▶ ISBN-13: 978-0-321-92416-2

Online Instructor's Solutions Manual (downloadable)

▶ By Garret J. Etgen, University of Houston

▶ This manual contains detailed solutions to all even-numbered section problems.

▶ Available in MyMathLab or through http://www.pearsonhighered.com/educator.

Mini Lectures (downloadable)

▶ By Salvatore Sciandra,
Niagara County Community College

▶ Mini Lectures are provided for the teaching assistant, adjunct, part-time or even full-time instructor for lecture preparation by providing learning objectives, examples (and answers) not found in the text, and teaching notes.

▶ Available in MyMathLab or through http://www.pearsonhighered.com/educator.

PowerPoint® Lecture Slides

▶ These slides present key concepts and definitions from the text. They are available in MyMathLab or at http://www.pearsonhighered.com/educator.

Technology Resources

MyMathLab® Online Course
(access code required)

MyMathLab delivers **proven results** in helping individual students succeed.

▶ MyMathLab has a consistently positive impact on the quality of learning in higher education math instruction. MyMathLab can be successfully implemented in any environment—lab based, hybrid, fully online, traditional—and demonstrates the quantifiable difference that integrated usage has on student retention, subsequent success, and overall achievement.

▶ MyMathLab's comprehensive online gradebook automatically tracks your students' results on tests, quizzes, homework, and in the study plan. You can use the gradebook to quickly intervene if your students have trouble or to provide positive feedback on a job well done. The data within MyMathLab is easily exported to a variety of spreadsheet programs, such as Microsoft Excel. You can determine which points of data you want to export and then analyze the results to determine success.

MyMathLab provides **engaging experiences** that personalize, stimulate, and measure learning for each student.

▶ **Personalized Learning:** MyMathLab offers two important features that support adaptive learning—personalized homework and the adaptive study plan. These features allow your students to work on what they need to learn when it makes the most sense, maximizing their potential for understanding and success.

▶ **Exercises:** The homework and practice exercises in MyMathLab are correlated to the exercises in the textbook, and they regenerate algorithmically to give students unlimited opportunity for practice and mastery. The software offers immediate, helpful feedback when students enter incorrect answers.

▶ **Chapter-Level, Just-in-Time Remediation:** The MyMathLab course for these texts includes a short diagnostic, called Getting Ready, prior to each chapter to assess students' prerequisite knowledge. This diagnostic can then be tied to personalized homework so that each student receives a homework assignment specific to his or her prerequisite skill needs.

▶ **Multimedia Learning Aids:** Exercises include guided solutions, sample problems, animations, videos, and eText access for extra help at the point of use.

And, MyMathLab comes from an **experienced partner** with educational expertise and an eye on the future.

▶ Knowing that you are using a Pearson product means that you are using quality content. That means that our eTexts are accurate and our assessment tools work. It means we are committed to making MyMathLab as accessible as possible. MyMathLab is compatible with the JAWS 12/13 screen reader, and enables multiple-choice and free-response problem types to be read and interacted with via keyboard controls and math notation input. More information on this functionality is available at http://mymathlab.com/accessibility.

▶ Whether you are just getting started with MyMathLab or you have a question along the way, we're here to help you learn about our technologies and how to incorporate them into your course.

▶ To learn more about how MyMathLab combines proven learning applications with powerful assessment and continuously adaptive capabilities, visit www.mymathlab.com or contact your Pearson representative.

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These new Ready-to-Go courses provide students with all the same great MyMathLab features but make it easier for instructors to get started. Each course includes preassigned homework and quizzes to make creating a course even simpler. In addition, these prebuilt courses include a course-level Getting Ready diagnostic that helps pinpoint student weaknesses in prerequisite skills. Ask your Pearson representative about the details for this particular course or to see a copy of this course.

MyLabsPlus®

MyLabsPlus combines proven results and engaging experiences from MyMathLab® and MyStatLab™ with convenient management tools and a dedicated services team. Designed to support growing math and statistics programs, it includes additional features such as

▶ **Batch Enrollment:** Your school can create the login name and password for every student and instructor, so everyone can be ready to start class on the first day. Automation of this process is also possible through integration with your school's Student Information System.

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▶ **Advanced Reporting:** MyLabsPlus advanced reporting allows instructors to review and analyze students' strengths and weaknesses by tracking their performance on tests, assignments, and tutorials. Administrators can review grades and assignments across all courses on your MyLabsPlus campus for a broad overview of program performance.

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MathXL is the homework and assessment engine that runs MyMathLab. (MyMathLab is MathXL plus a learning-management system.)

With MathXL, instructors can

▶ Create, edit, and assign online homework and tests using algorithmically generated exercises correlated at the objective level to the textbook.

▶ Create and assign their own online exercises and import TestGen tests for added flexibility.

▶ Maintain records of all student work tracked in MathXL's online gradebook.

With MathXL, students can

▶ Take chapter tests in MathXL and receive personalized study plans and/or personalized homework assignments based on their test results.

▶ Use the study plan and/or the homework to link directly to tutorial exercises for the objectives they need to study.

▶ Access supplemental animations and video clips directly from selected exercises.

MathXL is available to qualified adopters. For more information, visit our website at www.mathxl.com or contact your Pearson representative.

TestGen®

TestGen (www.pearsoned.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple, but equivalent, versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions. The software and test bank are available for download from Pearson Education's online catalog.

Acknowledgments

In addition to the authors, many others are involved in the successful publication of a book. We wish to thank the following reviewers:

Diagnostic Prerequisite Test

Work all of the problems in this self-test without using a calculator. Then check your work by consulting the answers in the back of the book. Where weaknesses show up, use the reference that follows each answer to find the section in the text that provides the necessary review.

1. Replace each question mark with an appropriate expression that will illustrate the use of the indicated real number property:

   (A) Commutative $(\,\cdot\,)$: $x(y + z) = ?$

   (B) Associative $(+)$: $2 + (x + y) = ?$

   (C) Distributive: $(2 + 3)x = ?$

Problems 2–6 refer to the following polynomials:

   (A) $3x - 4$ (B) $x + 2$

   (C) $2 - 3x^2$ (D) $x^3 + 8$

2. Add all four.

3. Subtract the sum of (A) and (C) from the sum of (B) and (D).

4. Multiply (C) and (D).

5. What is the degree of each polynomial?

6. What is the leading coefficient of each polynomial?

In Problems 7 and 8, perform the indicated operations and simplify.

7. $5x^2 - 3x[4 - 3(x - 2)]$

8. $(2x + y)(3x - 4y)$

In Problems 9 and 10, factor completely.

9. $x^2 + 7x + 10$

10. $x^3 - 2x^2 - 15x$

11. Write 0.35 as a fraction reduced to lowest terms.

12. Write $\dfrac{7}{8}$ in decimal form.

13. Write in scientific notation:

   (A) 4,065,000,000,000   (B) 0.0073

14. Write in standard decimal form:

   (A) 2.55×10^8 (B) 4.06×10^{-4}

15. Indicate true (T) or false (F):

   (A) A natural number is a rational number.

   (B) A number with a repeating decimal expansion is an irrational number.

16. Give an example of an integer that is not a natural number.

In Problems 17–24, simplify and write answers using positive exponents only. All variables represent positive real numbers.

17. $6(xy^3)^5$

18. $\dfrac{9u^8v^6}{3u^4v^8}$

19. $(2 \times 10^5)(3 \times 10^{-3})$

20. $(x^{-3}y^2)^{-2}$

21. $u^{5/3}u^{2/3}$

22. $(9a^4b^{-2})^{1/2}$

23. $\dfrac{5^0}{3^2} + \dfrac{3^{-2}}{2^{-2}}$

24. $(x^{1/2} + y^{1/2})^2$

In Problems 25–30, perform the indicated operation and write the answer as a simple fraction reduced to lowest terms. All variables represent positive real numbers.

25. $\dfrac{a}{b} + \dfrac{b}{a}$

26. $\dfrac{a}{bc} - \dfrac{c}{ab}$

27. $\dfrac{x^2}{y} \cdot \dfrac{y^6}{x^3}$

28. $\dfrac{x}{y^3} \div \dfrac{x^2}{y}$

29. $\dfrac{\dfrac{1}{7 + h} - \dfrac{1}{7}}{h}$

30. $\dfrac{x^{-1} + y^{-1}}{x^{-2} - y^{-2}}$

31. Each statement illustrates the use of one of the following real number properties or definitions. Indicate which one.

Commutative $(+, \cdot)$	Associative $(+, \cdot)$	Distributive
Identity $(+, \cdot)$	Inverse $(+, \cdot)$	Subtraction
Division	Negatives	Zero

   (A) $(-7) - (-5) = (-7) + [-(-5)]$

   (B) $5u + (3v + 2) = (3v + 2) + 5u$

   (C) $(5m - 2)(2m + 3) = (5m - 2)2m + (5m - 2)3$

   (D) $9 \cdot (4y) = (9 \cdot 4)y$

   (E) $\dfrac{u}{-(v - w)} = \dfrac{u}{w - v}$

   (F) $(x - y) + 0 = (x - y)$

32. Round to the nearest integer:

   (A) $\dfrac{17}{3}$ (B) $-\dfrac{5}{19}$

33. Multiplying a number x by 4 gives the same result as subtracting 4 from x. Express as an equation, and solve for x.

34. Find the slope of the line that contains the points $(3, -5)$ and $(-4, 10)$.

35. Find the x and y coordinates of the point at which the graph of $y = 7x - 4$ intersects the x axis.

36. Find the x and y coordinates of the point at which the graph of $y = 7x - 4$ intersects the y axis.

In Problems 37 and 38, factor completely.

37. $x^2 - 3xy - 10y^2$

38. $6x^2 - 17xy + 5y^2$

In Problems 39–42, write in the form $ax^p + by^q$ where a, b, p, and q are rational numbers.

39. $\dfrac{3}{x} + 4\sqrt{y}$ **40.** $\dfrac{8}{x^2} - \dfrac{5}{y^4}$

41. $\dfrac{2}{5x^{3/4}} - \dfrac{7}{6y^{2/3}}$ **42.** $\dfrac{1}{3\sqrt{x}} + \dfrac{9}{\sqrt[3]{y}}$

In Problems 43 and 44, write in the form $a + b\sqrt{c}$ where a, b, and c are rational numbers.

43. $\dfrac{1}{4 - \sqrt{2}}$ **44.** $\dfrac{5 - \sqrt{3}}{5 + \sqrt{3}}$

In Problems 45–50, solve for x.

45. $x^2 = 5x$

46. $3x^2 - 21 = 0$

47. $x^2 - x - 20 = 0$

48. $-6x^2 + 7x - 1 = 0$

49. $x^2 + 2x - 1 = 0$

50. $x^4 - 6x^2 + 5 = 0$

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A LIBRARY OF ELEMENTARY FUNCTIONS

1

Functions and Graphs

Introduction

The function concept is one of the most important ideas in mathematics. The study of mathematics beyond the elementary level requires a firm understanding of a basic list of elementary functions, their properties, and their graphs. See the inside back cover of this book for a list of the functions that form our library of elementary functions. Most functions in the list will be introduced to you by the end of Chapter 1. For example, in Section 1.3 you will learn how to apply quadratic functions to model the effect of tire pressure on mileage (see Problems 73 and 75 on page 48).

1.1 Functions

After a brief review of the Cartesian (rectangular) coordinate system in the plane and graphs of equations, we discuss the concept of function, one of the most important ideas in mathematics.

Cartesian Coordinate System

Recall that to form a **Cartesian** or **rectangular coordinate system**, we select two real number lines—one horizontal and one vertical—and let them cross through their origins as indicated in the figure below. Up and to the right are the usual choices for the positive directions. These two number lines are called the **horizontal axis** and the **vertical axis**, or, together, the **coordinate axes**. The horizontal axis is usually referred to as the *x* **axis** and the vertical axis as the *y* **axis**, and each is labeled accordingly. The coordinate axes divide the plane into four parts called **quadrants**, which are numbered counterclockwise from I to IV (see the figure).
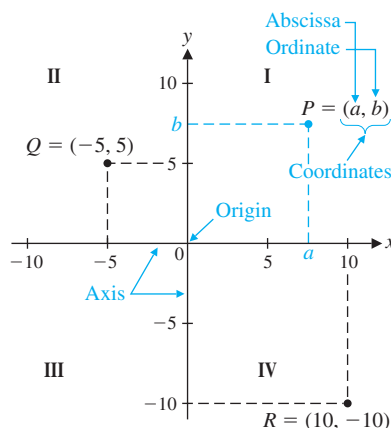
The Cartesian (rectangular) coordinate system

Now we want to assign *coordinates* to each point in the plane. Given an arbitrary point *P* in the plane, pass horizontal and vertical lines through the point (see figure). The vertical line will intersect the horizontal axis at a point with coordinate *a*, and the horizontal line will intersect the vertical axis at a point with coordinate *b*. These two numbers, written as the **ordered pair** (a, b) form the **coordinates** of the point *P*. The first coordinate, *a*, is called the **abscissa** of *P*; the second coordinate, *b*, is called the **ordinate** of *P*. The abscissa of *Q* in the figure is -5, and the ordinate of *Q* is 5. The coordinates of a point can also be referenced in terms of the axis labels. The *x* **coordinate** of *R* in the figure is 10, and the *y* **coordinate** of *R* is -10. The point with coordinates $(0, 0)$ is called the **origin**.

The procedure we have just described assigns to each point *P* in the plane a unique pair of real numbers (a, b). Conversely, if we are given an ordered pair of real numbers (a, b), then, reversing this procedure, we can determine a unique point *P* in the plane. Thus,

> There is a one-to-one correspondence between the points in a plane and the elements in the set of all ordered pairs of real numbers.

This is often referred to as the **fundamental theorem of analytic geometry**.

Graphs of Equations

A solution to an equation in one variable is a number. For example, the equation $4x - 13 = 7$ has the solution $x = 5$; when 5 is substituted for *x*, the left side of the equation is equal to the right side.

A solution to an equation in two variables is an ordered pair of numbers. For example, the equation $y = 9 - x^2$ has the solution $(4, -7)$; when 4 is substituted for x and -7 is substituted for y, the left side of the equation is equal to the right side. The solution $(4, -7)$ is one of infinitely many solutions to the equation $y = 9 - x^2$. The set of all solutions of an equation is called the **solution set**. Each solution forms the coordinates of a point in a rectangular coordinate system. To **sketch the graph** of an equation in two variables, we plot sufficiently many of those points so that the shape of the graph is apparent, and then we connect those points with a smooth curve. This process is called **point-by-point plotting**.

EXAMPLE 1   Point-by-Point Plotting   Sketch the graph of each equation.

(A) $y = 9 - x^2$ (B) $x^2 = y^4$

SOLUTIONS

(A) Make up a table of solutions—that is, ordered pairs of real numbers that satisfy the given equation. For easy mental calculation, choose integer values for x.

x	-4	-3	-2	-1	0	1	2	3	4
y	-7	0	5	8	9	8	5	0	-7

After plotting these solutions, if there are any portions of the graph that are unclear, plot additional points until the shape of the graph is apparent. Then join all the plotted points with a smooth curve (Fig. 1). Arrowheads are used to indicate that the graph continues beyond the portion shown here with no significant changes in shape.
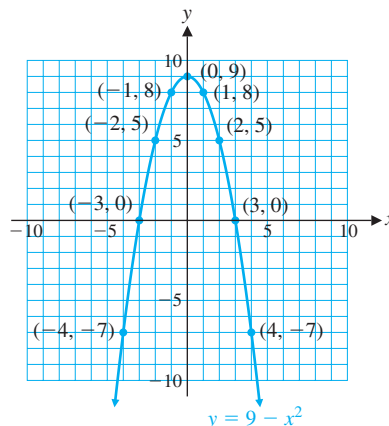
Figure 1 $y = 9 - x^2$

(B) Again we make a table of solutions—here it may be easier to choose integer values for y and calculate values for x. Note, for example, that if $y = 2$, then $x = \pm 4$; that is, the ordered pairs $(4, 2)$ and $(-4, 2)$ are both in the solution set.

x	± 9	± 4	± 1	0	± 1	± 4	± 9
y	-3	-2	-1	0	1	2	3

We plot these points and join them with a smooth curve (Fig. 2).
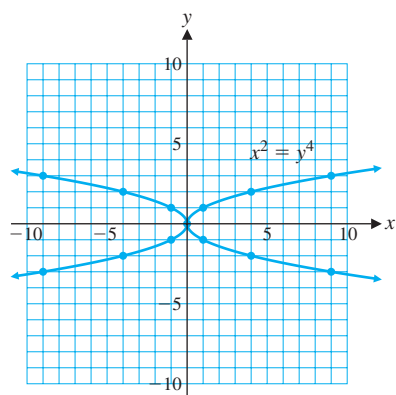
Matched Problem 1   Sketch the graph of each equation.

(A) $y = x^2 - 4$ (B) $y^2 = \dfrac{100}{x^2 + 1}$

Figure 2 $x^2 = y^4$

Explore and Discuss 1   To graph the equation $y = -x^3 + 3x$, we use point-by-point plotting to obtain

x	y
-1	-2
0	0
1	2
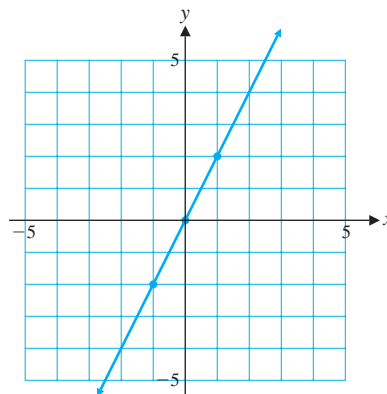
(A) Do you think this is the correct graph of the equation? Why or why not?

(B) Add points on the graph for $x = -2, -1.5, -0.5, 0.5, 1.5,$ and 2.

(C) Now, what do you think the graph looks like? Sketch your version of the graph, adding more points as necessary.

(D) Graph this equation on a graphing calculator and compare it with your graph from part (C).

(A)

(B)

Figure 3

The icon in the margin is used throughout this book to identify optional graphing calculator activities that are intended to give you additional insight into the concepts under discussion. You may have to consult the manual for your graphing calculator for the details necessary to carry out these activities. For example, to graph the equation in Explore and Discuss 1 on most graphing calculators, you must enter the equation (Fig. 3A) and the window variables (Fig. 3B).

As Explore and Discuss 1 illustrates, the shape of a graph may not be apparent from your first choice of points. Using point-by-point plotting, it may be difficult to find points in the solution set of the equation, and it may be difficult to determine when you have found enough points to understand the shape of the graph. We will supplement the technique of point-by-point plotting with a detailed analysis of several basic equations, giving you the ability to sketch graphs with accuracy and confidence.

Definition of a Function

Central to the concept of function is correspondence. You are familiar with correspondences in daily life. For example,

To each person, there corresponds an annual income.

To each item in a supermarket, there corresponds a price.

To each student, there corresponds a grade-point average.

To each day, there corresponds a maximum temperature.

For the manufacture of x items, there corresponds a cost.

For the sale of x items, there corresponds a revenue.

To each square, there corresponds an area.

To each number, there corresponds its cube.

One of the most important aspects of any science is the establishment of correspondences among various types of phenomena. Once a correspondence is known, predictions can be made. A cost analyst would like to predict costs for various levels of output in a manufacturing process; a medical researcher would like to know the

correspondence between heart disease and obesity; a psychologist would like to predict the level of performance after a subject has repeated a task a given number of times; and so on.

What do all of these examples have in common? Each describes the matching of elements from one set with the elements in a second set.

Consider Tables 1–3. Tables 1 and 2 specify functions, but Table 3 does not. Why not? The definition of the term *function* will explain.

Table 1

Domain	Range
Number	*Cube*
−2	−8
−1	−1
0	0
1	1
2	8

Table 2

Domain	Range
Number	*Square*
−2	4
−1	1
0	0
1	
2	

Table 3

Domain	Range
Number	*Square root*
0	0
	1
1	−1
	2
4	−2
	3
9	−3

DEFINITION Function

A **function** is a correspondence between two sets of elements such that to each element in the first set, there corresponds one and only one element in the second set.

The first set is called the **domain**, and the set of corresponding elements in the second set is called the **range**.

Tables 1 and 2 specify functions since to each domain value, there corresponds exactly one range value (for example, the cube of −2 is −8 and no other number). On the other hand, Table 3 does not specify a function since to at least one domain value, there corresponds more than one range value (for example, to the domain value 9, there corresponds −3 and 3, both square roots of 9).

Explore and Discuss 2   Consider the set of students enrolled in a college and the set of faculty members at that college. Suppose we define a correspondence between the two sets by saying that a student corresponds to a faculty member if the student is currently enrolled in a course taught by that faculty member. Is this correspondence a function? Discuss.

Functions Specified by Equations

Most of the functions in this book will have domains and ranges that are (infinite) sets of real numbers. The **graph** of such a function is the set of all points (x, y) in the Cartesian plane such that x is an element of the domain and y is the corresponding element in the range. The correspondence between domain and range elements is often specified by an equation in two variables. Consider, for example, the equation for the area of a rectangle with width 1 inch less than its length (Fig. 4). If x is the length, then the area y is given by

$$y = x(x - 1) \qquad x \geq 1$$

$x - 1$

x

Figure 4

For each **input** x (length), we obtain an **output** y (area). For example,

If $x = 5$,   then $y = 5(5 - 1) = 5 \cdot 4 = 20.$

If $x = 1$,   then $y = 1(1 - 1) = 1 \cdot 0 = 0.$

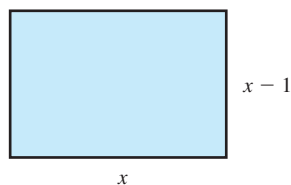If $x = \sqrt{5}$,   then $y = \sqrt{5}(\sqrt{5} - 1) = 5 - \sqrt{5}$

$\approx 2.7639.$

The input values are domain values, and the output values are range values. The equation assigns each domain value x a range value y. The variable x is called an *independent variable* (since values can be "independently" assigned to x from the domain), and y is called a *dependent variable* (since the value of y "depends" on the value assigned to x). In general, any variable used as a placeholder for domain values is called an **independent variable**; any variable that is used as a placeholder for range values is called a **dependent variable**.

When does an equation specify a function?

DEFINITION Functions Specified by Equations

If in an equation in two variables, we get exactly one output (value for the dependent variable) for each input (value for the independent variable), then the equation specifies a function. The graph of such a function is just the graph of the specifying equation.

If we get more than one output for a given input, the equation does not specify a function.

EXAMPLE 2 Functions and Equations Determine which of the following equations specify functions with independent variable x.

(A) $4y - 3x = 8$, x a real number   (B) $y^2 - x^2 = 9$, x a real number

SOLUTION

(A) Solving for the dependent variable y, we have

$$4y - 3x = 8$$
$$4y = 8 + 3x \tag{1}$$
$$y = 2 + \frac{3}{4}x$$

Since each input value x corresponds to exactly one output value $\left(y = 2 + \frac{3}{4}x\right)$, we see that equation (1) specifies a function.

(B) Solving for the dependent variable y, we have

$$y^2 - x^2 = 9$$
$$y^2 = 9 + x^2 \tag{2}$$
$$y = \pm\sqrt{9 + x^2}$$

Since $9 + x^2$ is always a positive real number for any real number x, and since each positive real number has two square roots,* then to each input value x there corresponds two output values $\left(y = -\sqrt{9 + x^2} \text{ and } y = \sqrt{9 + x^2}\right)$. For example, if $x = 4$, then equation (2) is satisfied for $y = 5$ and for $y = -5$. So equation (2) does not specify a function.

Matched Problem 2 Determine which of the following equations specify functions with independent variable x.

(A) $y^2 - x^4 = 9$, x a real number   (B) $3y - 2x = 3$, x a real number

Since the graph of an equation is the graph of all the ordered pairs that satisfy the equation, it is very easy to determine whether an equation specifies a function by examining its graph. The graphs of the two equations we considered in Example 2 are shown in Figure 5.

In Figure 5A, notice that any vertical line will intersect the graph of the equation $4y - 3x = 8$ in exactly one point. This shows that to each x value, there corresponds

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*Recall that each positive real number N has two square roots: $\sqrt{N}$, the principal square root; and $-\sqrt{N}$, the negative of the principal square root (see Appendix A, Section A.6).

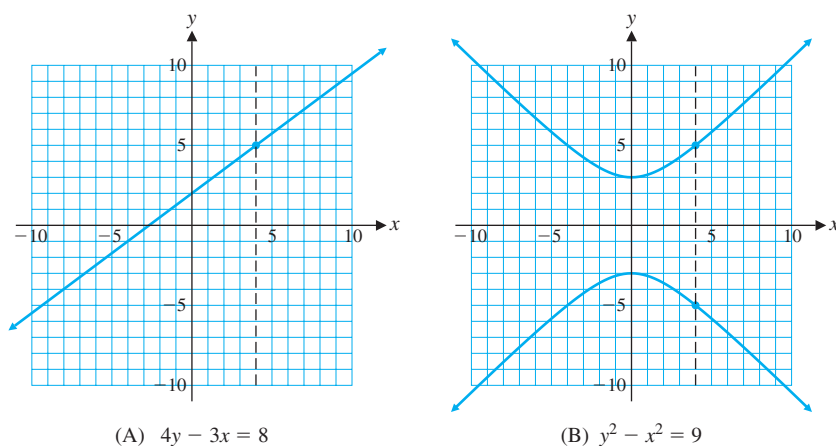(A) $4y - 3x = 8$ (B) $y^2 - x^2 = 9$

Figure 5

exactly one y value, confirming our conclusion that this equation specifies a function. On the other hand, Figure 5B shows that there exist vertical lines that intersect the graph of $y^2 - x^2 = 9$ in two points. This indicates that there exist x values to which there correspond two different y values and verifies our conclusion that this equation does not specify a function. These observations are generalized in Theorem 1.

> **THEOREM 1** Vertical-Line Test for a Function
>
> An equation specifies a function if each vertical line in the coordinate system passes through, at most, one point on the graph of the equation.
>   If any vertical line passes through two or more points on the graph of an equation, then the equation does not specify a function.

The function graphed in Figure 5A is an example of a *linear function*. The vertical-line test implies that equations of the form $y = mx + b$, where $m \neq 0$, specify functions; they are called **linear functions**. Similarly, equations of the form $y = b$ specify functions; they are called **constant functions**, and their graphs are horizontal lines. The vertical-line test implies that equations of the form $x = a$ do not specify functions; note that the graph of $x = a$ is a vertical line.

In Example 2, the domains were explicitly stated along with the given equations. In many cases, this will not be done. Unless stated to the contrary, we shall adhere to the following convention regarding domains and ranges for functions specified by equations:

> **If a function is specified by an equation and the domain is not indicated, then we assume that the domain is the set of all real-number replacements of the independent variable (inputs) that produce real values for the dependent variable (outputs). The range is the set of all outputs corresponding to input values.**

EXAMPLE 3 Finding a Domain Find the domain of the function specified by the equation $y = \sqrt{4 - x}$, assuming that x is the independent variable.

SOLUTION For y to be real, $4 - x$ must be greater than or equal to 0; that is,

$$4 - x \geq 0$$
$$-x \geq -4$$
$$x \leq 4 \qquad \text{Sense of inequality reverses when both sides are divided by } -1.$$

Domain: $x \leq 4$ (inequality notation) or $(-\infty, 4]$ (interval notation)

Matched Problem 3 Find the domain of the function specified by the equation $y = \sqrt{x - 2}$, assuming x is the independent variable.

Function Notation

We have seen that a function involves two sets, a domain and a range, and a correspondence that assigns to each element in the domain exactly one element in the range. Just as we use letters as names for numbers, now we will use letters as names for functions. For example, f and g may be used to name the functions specified by the equations $y = 2x + 1$ and $y = x^2 + 2x - 3$:

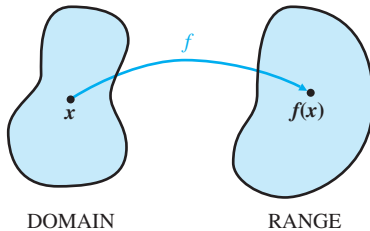$$f: \quad y = 2x + 1$$
$$g: \quad y = x^2 + 2x - 3 \tag{3}$$

If x represents an element in the domain of a function f, then we frequently use the symbol

$$f(x)$$

in place of y to designate the number in the range of the function f to which x is paired (Fig. 6). This symbol does *not* represent the product of f and x. The symbol $f(x)$ is read as "f of x," "f at x," or "the value of f at x." Whenever we write $y = f(x)$, we assume that the variable x is an independent variable and that both y and $f(x)$ are dependent variables.

DOMAIN   RANGE

Figure 6

Using function notation, we can now write functions f and g in equation (3) as

$$f(x) = 2x + 1 \quad \text{and} \quad g(x) = x^2 + 2x - 3$$

Let us find $f(3)$ and $g(-5)$. To find $f(3)$, we replace x with 3 wherever x occurs in $f(x) = 2x + 1$ and evaluate the right side:

$$f(x) = 2x + 1$$
$$f(\mathbf{3}) = 2 \cdot \mathbf{3} + 1$$
$$= 6 + 1 = 7 \quad \text{For input 3, the output is 7.}$$

Therefore,

$$f(3) = 7 \quad \text{The function } f \text{ assigns the range value 7 to the domain value 3.}$$

To find $g(-5)$, we replace each x by -5 in $g(x) = x^2 + 2x - 3$ and evaluate the right side:

$$g(x) = x^2 + 2x - 3$$
$$g(\mathbf{-5}) = (\mathbf{-5})^2 + 2(\mathbf{-5}) - 3$$
$$= 25 - 10 - 3 = 12 \quad \text{For input } -5, \text{ the output is 12.}$$

Therefore,

$$g(-5) = 12 \quad \text{The function } g \text{ assigns the range value 12 to the domain value } -5.$$

It is very important to understand and remember the definition of $f(x)$:

For any element x in the domain of the function f, the symbol $f(x)$ represents the element in the range of f corresponding to x in the domain of f. If x is an input value, then $f(x)$ is the corresponding output value. If x is an element that is not in the domain of f, then f is not defined at x and $f(x)$ does not exist.

EXAMPLE 4 Function Evaluation For $f(x) = 12/(x - 2)$, $g(x) = 1 - x^2$, and $h(x) = \sqrt{x - 1}$, evaluate:

(A) $f(6)$ (B) $g(-2)$ (C) $h(-2)$ (D) $f(0) + g(1) - h(10)$

SOLUTION

(A) $f(\mathbf{6}) = \dfrac{12}{\mathbf{6} - 2} * = \dfrac{12}{4} = 3$

(B) $g(\mathbf{-2}) = 1 - (\mathbf{-2})^2 = 1 - 4 = -3$

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*Dashed boxes are used throughout the book to represent steps that are usually performed mentally.